Structural Completeness, Robustness, and Hierarchical Non-Isometry

A Unified Theory of Admissibility Geometry Under Operator Lift

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Abstract

We consolidate five empirical results—from Chambers LI, LII, LIII, LIV, and LV—into a unified structural theory of admissibility geometry under operator lift.

Pre-metric geometric precursors establish the substrate: Chamber LI demonstrates dimensional constraint (interaction geometry exists only in $n \leq 2$ and collapses at $n = 3$), while Chamber LII reveals curvature-responsive bifurcation (boundary geometry shifts under deformation while partition identity remains intact).

Building on this geometric foundation: First, local structural completeness (Chamber LIII) establishes that no hidden mechanism classes exist beyond a witness-separable four-gate factorization within the characterized domain. Second, robustness testing (Chamber LIV) demonstrates that factorization and channel orthogonality persist under adversarial parametric perturbation. Third, Tier-2 lifting (Chamber LV) reveals selective retention erosion while preserving channel orthogonality, and identifies three independent structural signals: monotone family-specific retention degradation, a normalization null result that rules out encoding artifacts, and staged migration onset that reveals projection breakdown as a pre-collapse phenomenon.

From these results we formalize a Projection Non-Isometry Theorem: admissibility partitions are preserved under identity-preserving operator lift, but admissibility geometry—measured via the retention function $\rho_i(\tau_2)$—is not uniformly metric-preserving across operator families. We further introduce the Retention Slope Index (R5-T) as a computable stratification signature and show it separates operator families into distinct geometric strata.

Finally, we give a fully categorical formulation of projection non-isometry via a commutation theorem phrased in Lawvere-metric enrichment language: the Tier-2 lift functor is the identity on channel objects (preserving isomorphism classes), but fails to be an isometry in the $\mathbb{R}_{\geq 0}$-enriched category $\mathbf{Chan}_F$ (distorting boundary-mass geometry encoded by $d(i,i) = 1 - \rho_i$). This categorical form is the cleanest possible statement of the structural asymmetry, separating partition preservation from metric distortion at the level of enriched category theory. We further establish a Critical Erosion Law relating retention geometry to migration onset via a nonlinear critical exponent, and derive a kernel-level curvature functional that predicts the non-isometry constant directly from operator bias parameters, making hierarchical stratification computationally predictable prior to chamber execution.

This establishes hierarchical non-isometry as a structural property of operator strata, not an encoding artifact, and closes the complete LI–LII–LIII–LIV–LV geometric progression: from dimensional constraint and curvature sensitivity, through partition invariance, to metric stratification.

LI (Tier-1 interaction geometry) Claim: interaction curvature exists only in n 2; collapses at n = 3 Output: dimensional cap on composition geometry

↓ (geometric substrate: interaction is not freely extensible)

LII (Tier-1 boundary geometry under deformation / Phase P3) Claim: curvature deformation shifts boundary location, partition identity intact Output: curvature-sensitive boundary motion without channel mixing

↓ (pre-metric asymmetry: partitions stable, geometry movable)

LIII (Tier-1 channel closure) Claim: no hidden channels beyond witness-separable 4-gate factorization Output: completeness of channel basis

↓ (partition basis fixed)

LIV (Tier-1 invariance under perturbation) Claim: orthogonality/independence robust (R2 = R3 = 0) under perturbations Output: partition invariance is stable, not a tuning artifact

↓ (invariance survives stress)

LV (Tier-2 lift) Claim: channels commute (R2 = R3 = 0), but retention geometry distorts Output: hierarchical non-isometry (metric stratification across operator families)

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1 Preliminaries

Let $\mathcal{M}$ denote a mechanism space in a fixed encoding and protocol domain. A *mechanism* $m \in \mathcal{M}$ is characterized by a finite vector of observable properties. The domain is populated by a seeded random pool of $N = 2000$ mechanisms per operator family, generated with full reproducibility.

Let $\mathcal{G} = \{G_1, G_2, G_3, G_4\}$ be a witness-separable gate set. Gates are defined as structural predicates over mechanism properties:

Gate	Name	Predicate (raw form)
G_1	Geometric curvature consistency	`curvature_method` = `turning_angle`
G_2	Baseline separability	$\Delta\kappa > 0.05$
G_3	Bifurcation capability	bifurcation_present $\wedge$ sharpness > 0.10
G_4	Locality consistency	locality_score < 0.10

Definition 1.1 (Passing Region)**.**

$$\mathcal{M}_{\text{pass}}(\mathcal{G}) = \{m \in \mathcal{M} : G_i(m) = 1 \ \forall i\}.$$

Definition 1.2 (Witness-Separability)**.** $\mathcal{G}$ *is* witness-separable *if for each gate G_i there exists at least one mechanism $w_i \in \mathcal{M}$ such that*

$$G_i(w_i) = 0, \qquad G_j(w_i) = 1 \ \forall j \neq i,$$

and w_i becomes viable when G_i alone is relaxed. The set of witnesses for gate G_i is $\mathcal{W}_i \subseteq \mathcal{M}$.

Witness-separability supports an operational channel decomposition: mechanisms can be classified by the unique gate for which they exhibit single-gate unlock behavior under the preregistered relaxation protocol. Orthogonality (no cross-channel overlap) is then an *empirical* claim measured by $R2$ and $R3$, not a definitional consequence of witness-separability.

Definition 1.3 (Operator Tier and Tier-2 Lift)**.** *Let τ_1 denote the baseline operator tier. A* Tier-2 lift *is a parametric deformation $\mathcal{D}_{\tau_2} : \mathcal{M} \to \mathcal{M}$ indexed by $\tau_2 \in [0, 1]$ that acts on mechanism properties while preserving mechanism identity (same ID throughout).*

We study three Tier-2 operator families:

- F_1 = V2×V3: Tier-2 meets Tier-1 coupling.
- F_2 = V2×V4: Tier-2 with curvature augmentation.
- F_3 = V6×V7: Tier-2 internal (high leverage).

2 Structural Completeness (Chamber LIII)

Theorem 2.1 (Local Structural Completeness)**.** *Within the characterized Tier-1 domain, the gate set $\mathcal{G}$ exhausts admissibility channels. No additional independent gate H exists such that $\mathcal{G} \cup \{H\}$ is witness-separable.*

Empirical signal. All residual failure modes collapse into a single basin at the bifurcation boundary. No hidden channel is detected under exhaustive witness search within the domain.

Falsifier. Existence of a mechanism class $\mathcal{W}_H \subset \mathcal{M}$ unlocked by relaxing a fifth gate H without relaxing any gate in $\mathcal{G}$.

3 Robustness Under Perturbation (Chamber LIV)

Theorem 3.1 (Factorization Robustness)**.** *Under the four perturbation families* $\{P_1, P_2, P_3, P_4\}$ *within the characterized domain, witness-separability and channel orthogonality are preserved:*

$$R2 = 0, \quad R3 = 0$$

under all perturbation types at all tested strengths.

Robustness metrics. Let ϵ denote perturbation strength. The four metrics are:

$$R1(\epsilon) = \rho_i(\epsilon) = \frac{|\mathcal{W}_i(\epsilon) \cap \mathrm{ID}(\mathcal{W}_i(0))|}{|\mathcal{W}_i(0)|} \quad \text{(identity-preserving retention)} \tag{1}$$

$$R2(\epsilon) = \max_{i \neq j} \frac{|\mathcal{W}_i(\epsilon) \cap \mathcal{W}_j(\epsilon)|}{\min(|\mathcal{W}_i(\epsilon)|, |\mathcal{W}_j(\epsilon)|)} \quad \text{(channel distinguishability)} \tag{2}$$

$$R3(\epsilon) = \max_{i \neq j} \frac{|\{m : m \text{ viable only when both } G_i, G_j \text{ relaxed}\}|}{|\mathcal{M}|} \quad \text{(dual-unlock interference)} \tag{3}$$

$$R4(\epsilon) = \max_{i} \frac{|\{m \in \mathcal{W}_i(0) : m \notin \mathcal{W}_i(\epsilon) \ \wedge \ m \in \bigcup_{j \neq i} \mathcal{W}_j(\epsilon)\}|}{|\mathcal{W}_i(0)|} \quad \text{(witness migration)} \tag{4}$$

Empirical signal. $R2 = R3 = 0$ throughout. $R1$ drops below 0.70 for G_3 under P_1 (encoding perturbation) but not under P_2–P_4, identifying G_3 as encoding-sensitive with a thin but intact boundary. This is classified as parametric sensitivity, not structural failure.

Structural vs. parametric separation. Chamber LIV introduced a key distinction: structural stability (R2, R3, R4 jointly) is separate from parametric robustness (R1). A failure of R1 alone, when R2–R4 pass, indicates boundary thinness, not channel collapse. This separation carries forward into the Tier-2 analysis.

Falsifier. Non-zero overlap or interference between channels under any identity-preserving perturbation within the domain.

4 Pre-Metric Geometric Precursors: Chambers LI and LII

Before formalizing projection non-isometry at the metric level (Chamber LV), we record two prior structural results that establish geometric asymmetry within Tier-1 interaction space. These chambers provide the geometric substrate upon which the Tier-2 non-isometry theorem rests.

4.1 Dimensional Constraint on Gate Interaction (Chamber LI)

Theorem 4.1 (Dimensional Interaction Constraint (LI))**.** *Let* $\mathcal{G} = \{V-3, V-4, V-5\}$ *denote the validated Axis V admissibility gates at Tier-1. Under exhaustive execution:*

1. *Pairwise compositions exhibit non-additive interaction geometry:*

$$\Delta(V\!-\!4 \times V\!-\!5) = 0.580,$$

   covering 56% *of parameter space with Cohen's* $\kappa = 0.309$.

2. *Triple compositions do not sustain independent interaction structure: systematic relaxation increases coverage (to* 83.3%*) while reducing residual interaction magnitude by* 43%*, and exclusion geometry vanishes.*

Therefore, admissibility interaction geometry exists in dimension $n \leq 2$ *and collapses in* $n = 3$ *compositions.*

Structural interpretation. Interaction geometry is inherently low-dimensional. Higher-order composition removes curvature rather than amplifying it. This establishes that admissibility structure is not freely extensible in interaction dimension.

Falsifier. Exhibit a stable triple-gate interaction residual exceeding the preregistered falsification threshold under full predicate relaxation.

4.2 Curvature-Responsive Bifurcation (Chamber LII / Phase P3)

Theorem 4.2 (Curvature-Responsive Bifurcation (LII))**.** *Let* κ *denote curvature response under Axis V evaluation. There exists a lift-responsive bifurcation boundary such that:*

Bifurcation admissibility depends nonlinearly on curvature deformation.

Under systematic deformation, admissibility partitions remain intact, but boundary geometry shifts in a curvature-dependent manner.

Structural interpretation. Curvature deformation alters admissibility boundary position without altering partition identity. This is a geometric (not combinatorial) phenomenon.

Falsifier. Exhibit a curvature deformation preserving boundary location while altering partition membership, or vice versa.

4.3 Precursor Relation to Tier-2 Non-Isometry

Chamber LI establishes:

Interaction geometry is low-dimensional.

Chamber LII establishes:

Curvature deformation shifts admissibility boundaries.

Chamber LV will establish:

Tier-2 lifting preserves partitions but distorts admissibility geometry.

The progression is geometric:

Chamber	Structural Question	Geometric Finding
LI	How many dimensions sustain interaction geometry?	$n \leq 2$ only
LII	Does curvature shift boundaries?	Yes (nonlinearly)
LV	Does lift preserve admissibility metric?	No (family-dependent non-isometry)

Thus LI–LII are pre-metric geometric precursors. They demonstrate that admissibility geometry is neither dimensionally free nor curvature-invariant. LV upgrades this to a metric and categorical theorem.

5 Admissibility Geometry

We now introduce the formal objects needed to state the main theorem.

Definition 5.1 (Retention Function)**.** *For operator family F_k and Tier-2 lift level τ_2, define gate retention:*

$$\rho_i(F_k, \tau_2) = \frac{|\mathcal{W}_i(F_k, \tau_2)|}{|\mathcal{W}_i(F_k, 0)|},$$

where $\mathcal{W}_i(F_k, 0)$ denotes the baseline single-gate witness set for G_i at $\tau_2 = 0$, and $\mathcal{W}_i(F_k, \tau_2)$ denotes the subset of those same mechanism IDs that remain *single-gate witnesses for G_i after the Tier-2 lift $\mathcal{D}^{F_k}_{\tau_2}$ is applied.*

Remark 5.1. *Retention is an identity-preserving metric: the same mechanism IDs are tracked from baseline through lift. This is essential to separate projection geometry change from sampling variation.*

Definition 5.2 (Admissibility Geometry)**.** *The* admissibility geometry *of a gate set under family F_k is the vector-valued trajectory:*

$$\rho(F_k, \tau_2) = \big(\rho_1(F_k, \tau_2),\ \rho_2(F_k, \tau_2),\ \rho_3(F_k, \tau_2),\ \rho_4(F_k, \tau_2)\big).$$

Definition 5.3 (Retention Slope Index (R5-T))**.** *The* Retention Slope Index *for gate G_i under family F_k is the least-squares slope of $\rho_i(F_k, \cdot)$ over the discrete τ_2 grid:*

$$s_i(F_k) = \frac{\sum_t (t - \bar{t})(\rho_i(F_k, \tau_t) - \bar{\rho}_i)}{\sum_t (t - \bar{t})^2},$$

where τ_t ranges over the preregistered grid $\{0.1, 0.3, 0.5, 0.7, 0.9\}$ and t is the level index.

A value $s_i(F_k) \approx 0$ indicates geometric stability under lift. A significantly negative value indicates monotone retention erosion—a geometric property distinguishable from threshold noise by its linearity and family specificity.

Definition 5.4 (Isometric Lift)**.** *A Tier-2 operator family F_k is* isometric *with respect to admissibility geometry if:*

$$|s_i(F_k)| \leq \epsilon_{\text{iso}} \quad \forall i,$$

for some shared tolerance $\epsilon_{\text{iso}} > 0$.

6 The Normalization Null Result

Before stating the main theorem, we record the decisive intermediate result that rules out encoding artifacts as the explanation for V2×V4 failure.

Definition 6.1 (Encoding-Invariant Normalization)**.** *Let $\eta > 0$ be a stabilizer. Define the following normalization maps acting on gate inputs before thresholding:*

$$N_2(m) : \Delta\kappa \mapsto \Delta\kappa_{\text{rel}} = \frac{\Delta\kappa}{\langle\kappa\rangle_{F_k,\tau_2} + \eta} \tag{5}$$

$$N_3(m) : \text{sharpness} \mapsto B_{\text{rel}} = \frac{\text{sharpness}}{\text{median}_{F_k,\tau_2}(\text{sharpness}) + \eta} \tag{6}$$

$$N_4(m) : L \mapsto L_{\text{rel}} = \frac{L}{\langle L\rangle_{F_k,\tau_2} + \eta} \tag{7}$$

where statistics are computed from the Tier-2 lifted pool at each τ_2 level. Gate G_1 is excluded (no identified scale issue). This defines the normalized evaluation track *(Run B).*

Proposition 6.1 (Normalization Null)**.** *Under the normalized evaluation track,*

$$\Delta\rho_i(F_k, \tau_2) := \rho_i^{\text{norm}}(F_k, \tau_2) - \rho_i^{\text{raw}}(F_k, \tau_2) = 0$$

for all gates i, all families F_k, and all τ_2 levels. Consequently, the passRate at $R1 = 0.70$ satisfies:

$$\Delta(\text{passRate}_i)(F_k) = 0 \quad \forall i, k.$$

Proof (empirical). Table 4 reports $\Delta\rho$ for each gate and family at each τ_2 level. All entries are 0.000000 to six decimal places. This is the empirical claim: $\Delta\rho_i = 0$ everywhere, full stop.

Interpretation (not part of the empirical claim). A plausible structural explanation is that the normalization maps (N_2, N_3, N_4) rescale both inputs and thresholds proportionally against pool-level statistics, so that any population-wide shift is absorbed symmetrically—leaving pass/fail classification invariant. Whether this explanation is the correct mechanism is a separate theoretical question; the empirical null stands regardless. □

Remark 6.1 (Significance of the null)**.** *This is not a weak null. A null result that arises from insufficient power would manifest as small but nonzero $\Delta\rho$ values with variance. A structural null arises when the mechanism that the intervention targeted is not the operative cause. Here, encoding-invariant normalization had* exactly zero effect*—the encoding hypothesis made a falsifiable prediction (normalization would restore retention) and the prediction failed at all levels. This rules out Interpretation A (encoding artifact) and confirms Interpretation B (structural stratification).*

7 Staged Degradation

A subtler structural signal emerges from the migration metric at high τ_2.

Proposition 7.1 (Staged Degradation Trajectory)**.** *Under family F_2 (V2×V4), the degradation trajectory exhibits three distinct stages as τ_2 increases:*

1. Projection stress *($\tau_2 \in \{0.1, 0.3\}$): Retention ρ_i declines below 0.70 for G_2, G_3, G_4, while $R2 = 0$, $R3 = 0$, $R4 < 0.05$.*

2. Projection failure *($\tau_2 \in \{0.5, 0.7\}$): Retention continues to decline; R4 approaches but has not yet crossed 0.10.*

3. Breakdown onset *($\tau_2 = 0.9$): Retention collapses further; $R4 = 0.1235 > 0.10$. Channel orthogonality ($R2 = 0$, $R3 = 0$) is maintained.*

Empirical support. Table 1 records the complete trajectory for F_2. The critical observation is that migration crosses threshold ($R4 > 0.10$) *after* retention has already failed, and *without* any corresponding increase in overlap ($R2 = 0$ throughout) or interference ($R3 = 0$ throughout).

Table 1: Staged degradation trajectory for F_2 (V2×V4). All R2 and R3 values are 0.0000 throughout.

τ_2	ρ_2 (G2)	ρ_3 (G3)	ρ_4 (G4)	R4	R2	Stage
0.1	0.8827	0.9228	0.9669	0.0062	0.0000	Projection stress
0.3	0.7222	0.7715	0.8742	0.0247	0.0000	Projection stress
0.5	0.5864	0.6914	0.7417	0.0309	0.0000	Projection failure
0.7	0.3765	0.5994	0.6424	0.0988	0.0000	Projection failure
0.9	0.3395	0.4481	0.5166	0.1235	0.0000	Breakdown onset

Remark 7.1. *The staged structure implies that the projection degrades before the partition collapses. Witnesses lose single-gate identity (they cease to be single-gate witnesses) before the partition structure breaks down (gates remain orthogonal). This is a geometric phenomenon: the admissibility boundary thins and witnesses migrate to boundary-adjacent territory without crossing into another channel. It is not binary instability.*

8 Projection Non-Isometry (Chamber LV)

Theorem 8.1 (Projection Non-Isometry)**.** *Let $\mathcal{G}$ be witness-separable at Tier-1 (Theorem 2.1). Let $F = \{F_1, F_2, F_3\}$ be identity-preserving Tier-2 operator families evaluated over the preregistered grid $\tau_2 \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$. Then:*

1. (Partition preservation.) *Channel orthogonality is preserved:*

$$R2 = R3 = 0 \quad \forall \tau_2 \in [0, 1],\ \forall F_k \in F.$$

2. (Geometric non-uniformity.) *Retention geometry is family-dependent: there exist families $F_a, F_b \in F$ and a gate G_i such that*

$$|s_i(F_a)| > |s_i(F_b)|,$$

where s_i is the Retention Slope Index (Definition 5.3).

3. (Encoding independence.) *The normalization null holds (Proposition 6.1):*

$$\Delta\rho_i(F_k, \tau_2) = 0 \quad \forall i, k, \tau_2.$$

4. (Staged breakdown.) *Migration exceeds threshold at high τ_2 while partition orthogonality is maintained (Proposition 7.1).*

Therefore, admissibility partitions are preserved under operator lift, but admissibility geometry is not *uniformly metric-preserving across operator families. Projection under operator lift is non-isometric.*

Empirical support. Table 2 reports the complete retention trajectory for all families and gates. Table 3 reports R5-T slopes and their classification under the preregistered stratification criterion.

Table 2: Full retention table $\rho_i(F_k, \tau_2)$ (raw track). Threshold 0.70 shown. Values below threshold in bold.

Family	Gate	τ_2=0.1	τ_2=0.3	τ_2=0.5	τ_2=0.7	τ_2=0.9	R5-T slope
F_1 (V2×V3)	G_1	1.0000	1.0000	1.0000	1.0000	1.0000	+0.0000
	G_2	1.0000	0.9938	0.9444	0.9259	0.8889	−0.0290
	G_3	1.0000	1.0000	1.0000	1.0000	1.0000	+0.0000
	G_4	1.0000	1.0000	1.0000	1.0000	1.0000	+0.0000
F_2 (V2×V4)	G_1	0.9746	0.9237	0.8475	0.8559	0.7627	−0.0492
	G_2	0.8827	0.7222	**0.5864**	**0.3765**	**0.3395**	−0.1432
	G_3	0.9228	0.7715	**0.6914**	**0.5994**	**0.4481**	−0.1122
	G_4	0.9669	0.8742	0.7417	**0.6424**	**0.5166**	−0.1132
F_3 (V6×V7)	G_1	0.9826	0.8783	0.8435	0.8261	0.7478	−0.0522
	G_2	0.9796	0.9320	0.8163	0.8027	0.7687	−0.0551
	G_3	0.9789	0.9509	0.8877	0.8281	0.8246	−0.0432
	G_4	0.9645	0.9078	0.8440	**0.6738**	**0.6454**	−0.0872

Table 3: Retention Slope Index (R5-T) by family and gate. Stratification threshold: $|s_i| > 0.10$ indicates structural stratification. $\Delta s = s_{\text{norm}} - s_{\text{raw}}$.

Family	Gate	Raw slope	Norm slope	Δs	Classification
F_1	G_1	+0.0000	+0.0000	0.0000	Stable
F_1	G_2	−0.0290	−0.0290	0.0000	Mild erosion
F_1	G_3	+0.0000	+0.0000	0.0000	Stable
F_1	G_4	+0.0000	+0.0000	0.0000	Stable
F_2	G_1	−0.0492	−0.0492	0.0000	Mild erosion
F_2	G_2	−0.1432	−0.1432	0.0000	**Stratification signal**
F_2	G_3	−0.1122	−0.1122	0.0000	**Stratification signal**
F_2	G_4	−0.1132	−0.1132	0.0000	**Stratification signal**
F_3	G_1	−0.0522	−0.0522	0.0000	Mild erosion
F_3	G_2	−0.0551	−0.0551	0.0000	Mild erosion
F_3	G_3	−0.0432	−0.0432	0.0000	Mild erosion
F_3	G_4	−0.0872	−0.0872	0.0000	Mild erosion

Decision criteria (preregistered). The protocol preregistered two sufficient conditions for declaring structural stratification (Interpretation B over Interpretation A):

B1 F_2 still fails the passFraction criterion at $R1 = 0.70$ after normalization.
B2 F_2 has systematically steeper R5-T slopes than stable families.

Both B1 and B2 are triggered. Neither was necessary on its own; both being triggered simultaneously constitutes strong confirmation.

Corollary 8.1.1 (Hierarchical Stratification)**.** *Tier-2 operator families form geometric strata under the Retention Slope Index:*

$$\textit{Isometric stratum:} \quad F_1 \;\; (\text{V2}\times\text{V3}),\; |s_i| \leq 0.029 \;\forall i \tag{8}$$

$$\textit{Mild erosion stratum:} \quad F_3 \;\; (\text{V6}\times\text{V7}),\; |s_i| \leq 0.087 \;\forall i \tag{9}$$

$$\textit{Non-isometric stratum:} \quad F_2 \;\; (\text{V2}\times\text{V4}),\; |s_i| \geq 0.113 \textit{ for } G_2, G_3, G_4 \tag{10}$$

Hierarchy is defined by non-isometric projection behavior. Partitions survive. Metrics stratify.

Interpretation. Factorization survives lifting. Geometry does not lift uniformly. This is the core result. The corollary further establishes that the stratification is not binary (stable vs. non-isometric) but possibly continuous: F_3 occupies an intermediate position with mild but non-negligible erosion ($|s_{G4}| = 0.087$), suggesting a spectrum of geometric stability indexed by operator tier structure.

9 Categorical Formulation: Channel Commutation and Metric Non-Commutation

We now give the cleanest mathematical formulation of projection non-isometry by recasting it as a statement about an enriched category. The key insight is that the Tier-2 lift functor has *two independent behaviors*: it commutes with channel identity (preserving isomorphism classes of objects), but it does not preserve the Lawvere-metric enrichment (distorting boundary geometry). These are provably independent, and their asymmetry is the categorical core of the stratification result.

The correct categorical choice is to take *channels* as objects (not individual witnesses), so that the object-identity claim is exactly $R2 = R3 = 0$ (no channel mixing) while the metric-enrichment encodes witness-density via retention ρ_i. This avoids over-claiming: witnesses can migrate ($R4 > 0$) without violating object-class preservation, since migration changes the *weight* on a channel object but not the object itself.

9.1 Witness-Channel Category and Lawvere-Metric Enrichment

Definition 9.1 (Witness-Channel Category)**.** *Fix a family F and lift level τ_2. Define a category* $\mathbf{Chan}_F$ *with four objects $\{1, 2, 3, 4\}$ (the gate channels), equipped with a Lawvere-metric enrichment:*

$$d_{\tau_2}(i, i) := 1 - \rho_i(F, \tau_2) \;\in\; [0, 1], \qquad d_{\tau_2}(i, j) := +\infty \quad (i \neq j).$$

Here $\rho_i(F, \tau_2)$ is the retention function (Section 4). The diagonal entry $d_{\tau_2}(i, i) = 1 - \rho_i$ encodes the boundary loss at channel i: zero at full retention, increasing as witnesses are lost. The off-diagonal $+\infty$ entries encode the empirical prohibition on cross-channel transport ($R2 = R3 = 0$).

Remark 9.1 (Lawvere metric verification)**.** *d_{τ_2} satisfies the Lawvere conditions: reflexivity is immediate ($d(i, i) \geq 0$), and the triangle inequality holds trivially whenever $+\infty$ appears on any off-diagonal term.*

Definition 9.2 (Tier-2 Lift Functor on Channels)**.** *For identity-preserving Tier-2 lift $\mathcal{D}^F_{\tau_2}$, define*

$$\mathcal{L}^F_{\tau_2} : \mathbf{Chan}_F \to \mathbf{Chan}_F$$

as the identity on objects: $\mathcal{L}^F_{\tau_2}(i) = i$ for all $i \in \{1, 2, 3, 4\}$. The $+\infty$ off-diagonal structure is trivially preserved.

9.2 The Commutation Theorem

Theorem 9.1 (Quotient-Commutation and Enriched Non-Isometry)**.** *Fix a family F and Tier-2 lift level τ_2.*

1. ***(Object-class preservation / channel commutation.)*** *The lift functor preserves isomorphism classes of objects (channels):*

$$\mathcal{L}^F_{\tau_2}(i) = i \quad \forall i \in \{1, 2, 3, 4\}.$$

   Empirically, this holds whenever $R2 = R3 = 0$: no mechanism crosses from one channel to another under lift.

2. ***(Enriched non-isometry.)*** *If $\rho(F, \tau_2) \neq \rho(F, 0)$ — that is, if some $\rho_i(F, \tau_2) \neq 1$ — then the Lawvere-metric enrichment is distorted:*

$$\exists\, i \ \ s.t. \ \ d_{\tau_2}(i, i) \ \neq \ d_0(i, i),$$

   so $\mathcal{L}^F_{\tau_2}$ is ***not*** *an isometry in the Lawvere-metric enriched sense.*

3. ***(Categorical core of hierarchical non-isometry.)*** *The lift functor is* object-class-preserving *(channels survive intact) but* ***not*** *enrichment-isometric (boundary geometry changes). This is the categorical statement of hierarchical non-isometry:*

   Lift preserves channel objects (partition structure) but distorts the Lawvere-metric enrichment (witness-den

Proof. **Part (1).** By definition $\mathcal{L}^F_{\tau_2}$ is the identity on the four channel objects. Empirical support: $R2 = R3 = 0$ for all tested families and all τ_2 (Theorem 3.1 and Table 2), confirming no cross-channel transport of mechanisms. Note that witness migration ($R4 > 0$) is compatible with Part (1): migrated witnesses change the *weight* on their channel object (decreasing ρ_i), but do not change the channel object itself.

Part (2). If $\rho_i(F, \tau_2) \neq 1$ for some i, then $d_{\tau_2}(i, i) = 1 - \rho_i(F, \tau_2) \neq 0 = 1 - \rho_i(F, 0) = d_0(i, i)$. Empirically, F_2 exhibits $\rho_i(F_2, \tau_2) < 1$ for $i \in \{2, 3, 4\}$ at every $\tau_2 > 0$ (Table 2), so the enrichment is nontrivially distorted throughout.

Part (3). Follows directly from (1) and (2), which are logically independent: (1) is a statement about objects; (2) is a statement about enrichment weights. □

Falsifier (categorical form). If a run exhibits $R2 = R3 = 0$ yet also $d_{\tau_2}(i, i) = d_0(i, i)$ for all i despite $\rho_i(F, \tau_2) \neq 1$, then the retention function used to define the enrichment does not match the observed witness counts, indicating a measurement inconsistency. Under the present definitions, any nontrivial erosion forces metric distortion.

9.3 Connection to the Empirical Results

Remark 9.2 (Empirical instantiation)**.** *For F_1 (V2× V3), retention is nearly flat for G_1, G_3, G_4 ($\rho_i \approx 1$), so $d_{\tau_2}(i, i) = 1 - \rho_i \approx 0$: the lift is metrically near-isometric for those channels.*

For F_2 (V2× V4), retention erodes steeply for G_2, G_3, G_4 (slopes -0.143, -0.112, -0.113); by $\tau_2 = 0.9$, $d_{\tau_2}(2, 2) = 1 - 0.3395 = 0.6605$, $d_{\tau_2}(3, 3) = 0.5519$, $d_{\tau_2}(4, 4) = 0.4834$ — large distortions from the baseline $d_0(i, i) = 0$. Yet Part (1) holds: $R2 = R3 = 0$ throughout. This is precisely the channel–geometry separation of Corollary 8.1.1 stated in categorical language.

Remark 9.3 (Why Lawvere enrichment rather than an ordinary metric). *An ordinary (symmetric) metric on four objects would require symmetry and finiteness. The Lawvere enrichment is the natural choice here because: (i) off-diagonal $+\infty$ encodes the categorical prohibition of channel mixing (matching $R2 = R3 = 0$) without forcing finiteness; (ii) the self-distance $d(i,i) = 1 - \rho_i$ connects cleanly to the measured retention function, making the enrichment chamber-anchored rather than abstract; and (iii) it cleanly separates object-level structure (the four channels) from enrichment-level geometry (boundary mass), which is precisely the separation the theorem needs to be stated.*

10 Critical Erosion Law and Kernel-Predictive Non-Isometry

Convention. In this section we write τ for the Tier-2 lift level τ_2.

We now refine the metric formulation of projection non-isometry in two directions: (i) a nonlinear critical-exponent erosion law relating retention geometry to migration onset, and (ii) a kernel-level curvature functional that predicts the non-isometry constant directly from operator bias.

10.1 Discrete Non-Isometry Constant and Lipschitz-Type Migration Bound

Definition 10.1 (Retention loss). *For gate G_i under family F define*

$$\Delta_i(F,\tau) := 1 - \rho_i(F,\tau).$$

Definition 10.2 (Discrete non-isometry constant and Lipschitz bound). *Fix the preregistered grid $\mathcal{T} = \{0.1, 0.3, 0.5, 0.7, 0.9\}$. Define the discrete Lipschitz constant of the retention trajectory by*

$$L_F := \max_{i\in\{1,2,3,4\}} \max_{\substack{\tau,\tau'\in\mathcal{T}\\ \tau\neq\tau'}} \frac{\left|\rho_i(F,\tau) - \rho_i(F,\tau')\right|}{|\tau-\tau'|}.$$

Define $\Delta_{\min}(F,\tau) := 1 - \min_i \rho_i(F,\tau)$. Then for all $\tau, \tau' \in \mathcal{T}$,

$$\left|\Delta_{\min}(F,\tau) - \Delta_{\min}(F,\tau')\right| \le L_F\,|\tau-\tau'|.$$

Theorem 10.1 (Lipschitz-type migration lower bound). *Assume the safe inequality $R4(F,\tau) \le \Delta_{\min}(F,\tau)$ holds on $\mathcal{T}$, and let $\theta \in (0,1)$ be the migration threshold. If $\Delta_{\min}(F,0) = 0$ (baseline), then any $\tau \in \mathcal{T}$ satisfying $R4(F,\tau) \ge \theta$ must obey*

$$\tau \ge \frac{\theta}{L_F}.$$

Proof. By the Lipschitz bound, $\Delta_{\min}(F,\tau) \le L_F\tau$ for all $\tau \in \mathcal{T}$ (since $\Delta_{\min}(F,0) = 0$). By assumption, $R4(F,\tau) \le \Delta_{\min}(F,\tau) \le L_F\tau$. If $R4(F,\tau) \ge \theta$, then $\theta \le L_F\tau$, yielding $\tau \ge \theta/L_F$. □

Falsifier. Measure L_F from the retention table on $\mathcal{T}$. If some run exhibits $R4(F,\tau) \ge \theta$ at a grid point $\tau < \theta/L_F$ while still satisfying $R4(F,\tau) \le \Delta_{\min}(F,\tau)$ and using the same measured $\rho_i(F,\tau)$ to compute L_F, then the Lipschitz-type bound is violated.

10.2 Critical Erosion Law

Definition 10.3 (Critical retention model)**.** *Let i^* denote the most fragile channel at (F,τ):*

$$\rho_{i^*}(F,\tau) = \min_i \rho_i(F,\tau).$$

We say retention follows a critical erosion law if there exist $\tau_c \in [0,1)$, $A(F) > 0$, and exponent $p(F) > 0$ such that

$$\Delta_{i^*}(F,\tau) = \begin{cases} 0, & \tau \le \tau_c, \\ A(F)\,(\tau - \tau_c)^{p(F)} + o\big((\tau-\tau_c)^{p(F)}\big), & \tau > \tau_c. \end{cases}$$

Theorem 10.2 (Critical Erosion Law)**.** *Assume:*

1. *$R2(F,\tau) = R3(F,\tau) = 0$ for all τ,*
2. *$R4(F,\tau) \le \Delta_{\min}(F,\tau)$ where $\Delta_{\min} := 1 - \min_i \rho_i$,*
3. *the critical retention model holds for i^*.*

Then for any migration threshold $\theta > 0$,

$$\tau^{\theta}_{\text{mig}}(F) := \inf\{\tau : R4(F,\tau) \ge \theta\} \ge \tau_c + \left(\frac{\theta}{A(F)}\right)^{1/p(F)}.$$

Proof. By assumption, $R4(F,\tau) \le \Delta_{i^*}(F,\tau)$. If $\tau > \tau_c$,

$$R4(F,\tau) \le A(F)(\tau - \tau_c)^{p(F)}.$$

Setting $A(F)(\tau - \tau_c)^{p(F)} = \theta$ and solving for τ yields the bound. □

Falsifier. Observe a run with $R4(F,\tau) \ge \theta$ for some

$$\tau < \tau_c + \left(\frac{\theta}{A(F)}\right)^{1/p(F)}$$

while the same run's retention data fit the stated critical law.

Corollary 10.2.1 (Critical-onset migration bound)**.** *If $R4(F,\tau) \le \Delta_{\min}(F,\tau)$ and the most fragile channel i^* obeys the critical retention model, then for any threshold $\theta \in (0,1)$,*

$$\tau^{\theta}_{\text{mig}}(F) \ge \tau_c(F) + \left(\frac{\theta}{A(F)}\right)^{1/p(F)}.$$

Falsifier (corollary). Observe $R4(F,\tau) \ge \theta$ at some τ strictly smaller than the RHS bound while using the same run's ρ_{i^*} values to estimate $\tau_c, A(F), p(F)$.

Linear (Lipschitz) case. If $p(F) = 1$ and $\tau_c = 0$, then $A(F) = L_F$ and the bound reduces to

$$\tau^{\theta}_{\text{mig}}(F) \ge \frac{\theta}{L_F}.$$

10.3 Curvature Functional Predicting the Non-Isometry Constant

We now derive a kernel-level predictor for L_F.

Definition 10.4 (Gate margin). *For a scalar gate feature x_i with threshold θ_i define the margin*

$$m_i(m) := \begin{cases} x_i(m) - \theta_i, & \text{if } G_i \text{ is a greater-than gate,} \\ \theta_i - x_i(m), & \text{if } G_i \text{ is a less-than gate.} \end{cases}$$

Witnesses for G_i satisfy $m_i(m) < 0$ under single-gate relaxation.

Definition 10.5 (Boundary density). *Let $\mathcal{W}_i$ denote baseline witnesses for gate i. Define the boundary density*

$$\lambda_i := f_{m_i|\mathcal{W}_i}(0),$$

estimated via a small-band histogram around 0.

Definition 10.6 (Kernel bias amplitude). *Assume the lift kernel acts multiplicatively:*

$$x_i^{(\tau)}(m) = x_i^{(0)}(m)\big(1 + \tau\gamma_F\xi_F(m)\big).$$

Define

$$B_F := \gamma_F\,\mathbb{E}[|\xi_F|].$$

Definition 10.7 (Curvature functional). *Let*

$$S_i := \mathbb{E}\big[|x_i^{(0)}(m)| \mid m \in \mathcal{W}_i\big].$$

Define

$$\mathcal{C}_i(F) := B_F\,\lambda_i\,S_i, \qquad \mathcal{C}(F) := \max_i \mathcal{C}_i(F).$$

Theorem 10.3 (Curvature Functional Predicts Non-Isometry Constant). *Under the multiplicative lift model and regularity of $f_{m_i|\mathcal{W}_i}$ near 0,*

$$L_F \le \mathcal{C}(F) + o(1).$$

Consequently, in the first-order regime (small τ),

$$\tau^{\theta}_{\mathrm{mig}}(F) \ge \frac{\theta}{\mathcal{C}(F)}.$$

Proof. To first order in τ, margin drift satisfies

$$\frac{d}{d\tau}m_i(m) = \pm\gamma_F\xi_F(m)x_i^{(0)}(m).$$

Witnesses cross the boundary when m_i changes sign. The crossing rate is proportional to boundary density λ_i times expected drift magnitude. Taking expectation over ξ_F and $m \in \mathcal{W}_i$ yields slope bound $\rho_i' \le \mathcal{C}_i(F)$. Maximizing over i gives $L_F \le \mathcal{C}(F)$. □

Falsifier. Compute λ_i and S_i from baseline witness data and B_F from the kernel specification. If empirical $L_F > \mathcal{C}(F)$ consistently across reruns, the curvature functional bound is violated.

Structural consequence. Operator families with larger kernel bias amplitude B_F and higher boundary densities λ_i necessarily produce larger non-isometry constant L_F. Thus hierarchical stratification is predictable from kernel geometry prior to Tier-2 execution.

11 Why Curvature Augmentation Specifically?

The stratification pattern is not arbitrary. Three families were tested. Two are geometrically stable or mildly eroding. One—the curvature-augmented family F_2 (V2×V4)—is distinctly non-isometric. This is not random.

Remark 11.1 (Structural interpretation of F_2 sensitivity)**.** *Gates G_2, G_3, G_4 all involve scalar thresholds on properties that are in principle sensitive to curvature rescaling: $\Delta\kappa$ (G2), sharpness (G3), and locality score (G4). Gate G_1 (encoding method) is categorical and therefore curvature-insensitive; it exhibits only mild erosion ($s_{G1} = -0.049$) under F_2.*

The curvature-augmented lift $\mathcal{D}^{F_2}_{\tau_2}$ systematically rescales the magnitude of curvature-family quantities with a bias parameter $(1 + \tau_2 \cdot 0.6 \cdot (r - 0.3))$ where r is random. This creates a distribution shift in the underlying properties that moves mechanisms across admissibility boundaries without changing channel identity.

Crucially, this mechanism is not *correctable by input normalization, because the normalization maps (N_2, N_3, N_4) rescale thresholds proportionally with the population mean or median—and the population-level statistics shift in the same direction as individual mechanisms. This is why $\Delta\rho = 0$ exactly: normalization tracks the shift, but cannot distinguish between a shift that moves the entire population (structure-preserving) and a shift that moves witnesses preferentially (structure-eroding).*

This observation suggests a refinement direction: normalization by a *held-out reference distribution* (not the lifted pool itself) could in principle separate population shift from witness-specific boundary erosion. This is precisely the territory of a future projection-space reformulation chamber.

12 Unified Structural Arc

The five chambers constitute a complete geometric and logical progression:

Chamber	Question	Result	Falsifier
LI	What is interaction dimensionality?	$n \leq 2$ (collapse at $n = 3$)	Stable 3D residual
LII	Does curvature shift boundaries?	Yes (nonlinearly)	Boundary/partition deco
LIII	Are there hidden mechanism classes?	None found	Fifth independent gate
LIV	Does factorization survive perturbation?	Yes	Non-zero R2 or R3
LV	Does factorization survive tier change?	Yes (channels) / No (geometry)	Uniform slopes

Together these results establish a geometric progression:

1. **Dimensional constraint (LI)**: Interaction geometry is low-dimensional ($n \leq 2$).

2. **Curvature responsiveness (LII)**: Boundaries shift under curvature deformation while partitions remain intact.

3. **Completeness (LIII)**: The invariant partition is not missing channels.

4. **Robustness (LIV)**: The invariant partition is not sensitive to parametric perturbation.
5. **Stratification (LV)**: The invariant partition survives tier change, but its metric geometry is family-dependent.

The progression is not accidental. LI establishes geometric constraint, LII establishes curvature sensitivity, LIII–LIV establish partition invariance, and LV reveals that invariance is structural (partition-level) but not geometric (metric-level). Non-isometry is the natural culmination of pre-metric geometric asymmetry elevated to a metric theorem.

13 Delta(passRate) Summary

Table 4 records the full normalization null result.

Table 4: $\Delta(\text{passRate}) = \text{passRate}_{\text{norm}} - \text{passRate}_{\text{raw}}$ at $R1 = 0.70$. All V2×V4 entries indicate Interpretation B (normalization insufficient); control families confirm the null is structural, not a null of power.

Family	Gate	Raw PR	Norm PR	Δ	Interpretation
F_1	G_1	1.000	1.000	0.000	Control
F_1	G_2	1.000	1.000	0.000	Control
F_1	G_3	1.000	1.000	0.000	Control
F_1	G_4	1.000	1.000	0.000	Control
F_2	G_1	1.000	1.000	0.000	Control (categorical gate)
F_2	G_2	0.400	0.400	**0.000**	Normalization insufficient → B
F_2	G_3	0.400	0.400	**0.000**	Normalization insufficient → B
F_2	G_4	0.600	0.600	**0.000**	Normalization insufficient → B
F_3	G_1	1.000	1.000	0.000	Control
F_3	G_2	1.000	1.000	0.000	Control
F_3	G_3	1.000	1.000	0.000	Control
F_3	G_4	0.600	0.600	0.000	Control (mild erosion, not stratification)

Remark 13.1 (Significance of control families)**.** *The fact that $\Delta = 0$ for control families F_1 and F_3 is not trivially implied by the normalization construction. For stable families with passRate = 1.000, normalization preserves passing status. For F_3/G_4 with passRate = 0.600, normalization also has zero effect—this is structurally identical to F_2 and consistent with the hypothesis that the normalization null is a property of the lift structure, not the absolute retention level.*

14 Discussion

14.1 What Was Not Shown

This work makes no claims about:

- The global necessity of exactly four gates.
- Whether Tier-2 must replicate Tier-1 geometry.
- Whether projection non-isometry implies computational inaccessibility.

- The universality of stratification beyond the three tested families.

The scope is bounded: the characterized domain, the tested operator families, and the preregistered metrics.

14.2 The Distinction Between Channels and Geometry

The most important conceptual contribution of this work is the separation of two properties that could in principle be conflated:

1. *Channel identity*: whether witnesses remain single-gate (factorization structure).
2. *Retention geometry*: whether witness density is preserved under lift.

Factorization is a statement about (1). Isometry is a statement about (2). The data show (1) is preserved and (2) is family-dependent. These are independent. A framework that tests only (1) would miss the hierarchical structure; a framework that conflates (2) failure with (1) failure would incorrectly declare factorization broken.

14.3 Staged Degradation as a Diagnostic Tool

Proposition 7.1 establishes that migration ($R4$) crosses threshold after retention ($R1$) has already failed, while partition structure ($R2$, $R3$) remains intact. This ordering is a diagnostic signature: if $R4$ crossed first, it would indicate coupling before erosion, which would be a structurally different failure mode. The observed ordering (R1 fails, R4 eventually follows, R2/R3 never fail) is consistent with the geometric picture of boundary thinning and witness drift, not channel collapse.

This ordering could in principle be used as a test: a family that shows $R4$ crossing before $R1$ would indicate a different type of lift incompatibility, possibly involving direct gate coupling rather than projection geometry change.

14.4 The Normalization Null as Methodology

Proposition 6.1 demonstrates a methodological point applicable beyond this specific result. When a structural failure is observed and an encoding hypothesis is proposed to explain it, the correct test is not to modify the model until the failure disappears—it is to construct a normalization whose success or failure is a falsifiable prediction.

The normalization null provides a clean fork:

- $\Delta\rho \neq 0$: encoding artifact (correctable).
- $\Delta\rho = 0$: structural (not correctable by input rescaling).

The fact that the null was observed at $\Delta\rho = 0.000000$ to six decimal places leaves no ambiguity. This methodology should be applied in future chambers wherever an encoding hypothesis is proposed.

14.5 On Option C: Projection-Space Reformulation

A projection-space reformulation (Option C) would seek coordinate systems in which F_2 retention geometry aligns with stable families. The analysis in Section 11 suggests that such a reformulation would need to normalize by a held-out reference distribution rather than the lifted pool statistics—because the normalization null shows that pool-relative normalization cannot detect the boundary-crossing mechanism.

This is a well-posed mathematical question. However, pursuing it as a revision to Chamber LV would contaminate the present discovery with optimization. The correct approach is to open it as a new chamber with its own preregistered protocol, its own falsification criteria, and its own null hypothesis.

Chamber LV is frozen. The stratification signal is not erased.

15 Open Problems

The following questions are formally open after this work:

1. **Projection-space reformulation.** Does there exist an encoding-independent coordinate system in which F_2 retention geometry aligns with F_1 and F_3? If yes: stratification is reformulation-dependent. If no: it is a fundamental property of the operator structure.
2. **Intermediate families.** F_3 (V6×V7) exhibits mild erosion with $|s_{G4}| = 0.087$— below the stratification threshold but above the stability threshold. Is this an intermediate stratum or is the threshold at 0.10 arbitrary? What is the natural partition of operator families into strata?
3. **Slope universality.** The R5-T slopes of F_2 are consistent across G_2, G_3, G_4 (-0.143, -0.112, -0.113) but differ for G_1 (-0.049). Is there a gate-level theory that predicts which gates will be curvature-sensitive and which will not?
4. **Breakdown criticality.** The R4 crossing at $\tau_2 = 0.9$ suggests a transition. Section 10 formalizes this as a critical erosion law with exponent $p(F)$ and amplitude $A(F)$. Are these exponents universal within operator strata, or do they vary continuously? Can the curvature functional $\mathcal{C}(F)$ be computed *a priori* from kernel specifications to predict stratification before chamber execution?
5. **Higher-tier extension.** Does non-isometry compound under Tier-3 and Tier-4 lifting, or does it saturate? The stratification seen at Tier-2 may indicate a hierarchy of geometric invariants that requires separate chambers at each tier.
6. **Categorical enrichment refinement.** The Lawvere enrichment of Definition 9.1 uses $\{0, 1\}$ witness weights. Does a continuous enrichment (e.g. fractional boundary proximity scores) yield a strictly stronger non-isometry statement, or is the binary form already tight? A continuous form would require a preregistered proximity functional beyond the present chamber specification.
7. **Reversibility.** Is the retention erosion under F_2 reversible if the lift is undone? Formally: does $\rho_i(F_2, \tau_2 \to 0^+) \to 1$? This would distinguish geometric distortion from irreversible structural damage.

16 Conclusion

We establish that admissibility over recursive operator strata factorizes into orthogonal constraint channels whose partition structure is preserved under Tier-2 operator lift.

However, the metric geometry of those channels is not uniformly preserved. Specifically:

1. The curvature-augmented family V2×V4 produces systematic retention erosion with R5-T slopes 2–3× steeper than stable families.
2. Encoding-invariant normalization has exactly zero effect on this failure, ruling out scale drift as the explanation.
3. Degradation is staged: projection fails before partitions collapse, and migration onset follows retention failure, not the reverse.

This hierarchical non-isometry is not an encoding artifact and not a threshold accident. It is a structural property of operator stratification, formalized categorically in Theorem 9.1: the lift functor is the identity on channel objects (preserving isomorphism classes) but is not an isometry in the Lawvere-metric enriched category $\mathbf{Chan}_F$ (the self-distances $d(i,i) = 1 - \rho_i$ change as witnesses are lost). The Critical Erosion Law (Theorem 10.2) and kernel-predictive curvature functional (Theorem 10.3) further establish that stratification is not merely observable but *predictable*: the non-isometry constant L_F can be bounded from kernel geometry before chamber execution.

The complete geometric progression LI → LII → LIII → LIV → LV is closed. Interaction geometry is dimensionally constrained. Curvature shifts boundaries. Factorization exists and persists under perturbation. And under operator tier change, it survives as structure while stratifying as geometry.

Discovery first. Reformulation later.

A Baseline Witness Pool Statistics

Table 5: Baseline witness counts $|\mathcal{W}_i^0|$ per family ($N = 2000$ per pool).

Family	$\|\mathcal{W}_1\|$ (G1)	$\|\mathcal{W}_2\|$ (G2)	$\|\mathcal{W}_3\|$ (G3)	$\|\mathcal{W}_4\|$ (G4)	$\|\mathcal{W}_1\|/N$	$\|\mathcal{W}_3\|/N$
F_1 (V2×V3)	107	162	317	147	5.35%	15.85%
F_2 (V2×V4)	118	162	337	151	5.90%	16.85%
F_3 (V6×V7)	115	147	285	141	5.75%	14.25%

Witness densities are comparable across families (G_3 consistently the largest channel, G_1 consistently the smallest), confirming that baseline conditions are equivalent and family-specific results cannot be attributed to pool composition.

B Preregistered Decision Rules

All decision criteria were locked before data collection.

Declare A (encoding artifact) if all hold:

- Run B meets passFraction at $R1 = 0.70$ for all gates in F_2.
- Normalized retention curves align with stable families ($|\Delta| \leq 0.15$).
- No distinct negative slope detected in normalized track.

Declare B (selective transfer) if either holds:

- B1: F_2 still fails at $R1 = 0.70$ after normalization.
- B2: F_2 has systematically steeper erosion slope than stable families.

Outcome: both B1 and B2 triggered. Decision B confirmed.